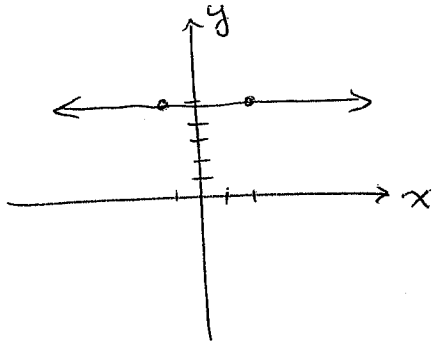
Math 45 chap 3 between 3.6 and 3.7

Vertical and Horizontal Lines

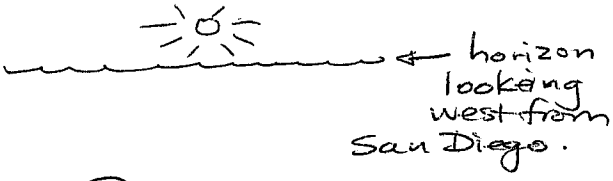
2 handouts
- summary graphs
- summary eqns

Objectives:
1) Recognize equation and graph of a vertical line
2) Recognize equation and graph of a horizontal line
3) Calculate once and memorize slope of vertical line.
4) Calculate one and memorize slope of horizontal line.
5) Write equations of vertical and horizontal lines given slope and point information.
6) Write equations of lines parallel or perpendicular to given vertical or horizontal lines.

① Plot (2,5) and (-1,5) and draw the line connecting them. Is this vertical or horizontal?



horizontal — based on the word "horizon"



② Calculate the slope of the line in ①.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-5}{-1-2}$$

$$= \frac{0}{-3}$$

$$m = 0$$

Memorize: All horizontal lines have slope = 0.

③ Write equation of line in ①.

LONG WAY: Point-slope $y - y_1 = m(x - x_1)$

$y - 5 = 0(x - 2)$

$y - 5 = 0$

$y = 5$

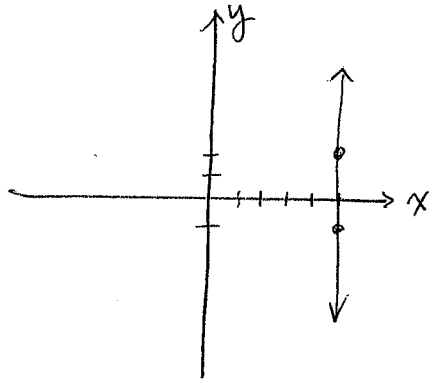
short cut: memorize: All horizontal lines have equations $y = \#$ (no x in equation)

All y-coords of pts on line are the same # in equation.

write $y =$ y-coord given

$y = 5$

④ Plot (3,2) and (3,−1) and draw the line connecting them. Is this line vertical or horizontal?



Vertical

⑤ Calculate the slope of the line in ④.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-(-1)}{3-3}$$

$$= \frac{3}{0}$$

m = undefined

Note: Some books or teachers call this "no slope."

Memorize: All vertical lines have undefined slope.

⑥ Write equation of line in ④.

* This CANNOT be done using point-slope! *

Memorize: All horizontal lines have equations $x = \#$ no y-in equation.

All x-coords of pts on line are the same # in equation.

write $x =$ x-coord given

$x = 3$

⑦ Write the equation of a horizontal line through $(-4,2)$.

horizontal $\Rightarrow$ $y = \#$ $\Rightarrow$ $y =$ ycoord $\Rightarrow$ $y = 2$

⑧ Write equation of a vertical line through $(-4,2)$.

vertical $\Rightarrow$ $x = \#$ $\Rightarrow$ $x =$ xcoord $\Rightarrow$ $x = -4$

(9) Write the equation of a line with undefined slope passing through $(7, -5)$.

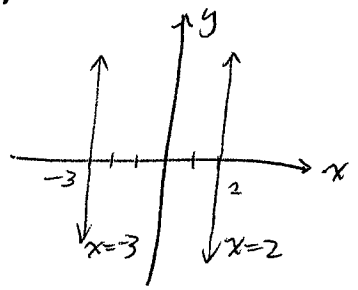
undefined slope $\Rightarrow$ vertical $\Rightarrow$ $x =$ x-coord $\Rightarrow$ $\boxed{x = 7}$.

(10) Write the equation of a line with zero slope passing through $(7, -5)$

zero slope $\Rightarrow$ horizontal $\Rightarrow$ $y =$ y coord $\Rightarrow$ $\boxed{y = -5}$

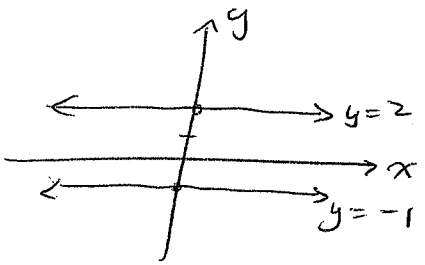
Graph the equations on the same axes. Are they parallel, perpendicular, or neither?

(11) $x = 2$
$x = -3$



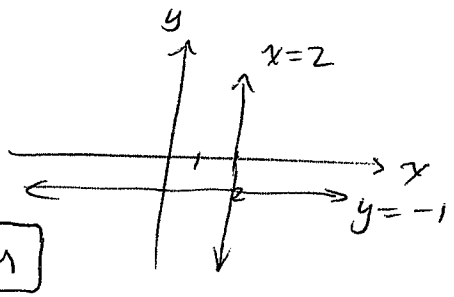
parallel

(12) $y = 2$
$y = -1$



parallel

(13) $x = 2$
$y = -1$



perpendicular

Memorize:

1) Vertical is parallel to vertical.
2) Horizontal is parallel to horizontal.
3) Vertical is perpendicular to horizontal.
4) Horizontal is perpendicular to vertical.

Without graphing, parallel, perpendicular, or neither?

(14) $x=5$
$y=3$
vertical $\perp$ horizontal $\Rightarrow$ **perpendicular**

(15) $y=2$
$y=-3$
horizontal $\parallel$ horizontal $\Rightarrow$ **parallel**

(16) $x=-1$
$x=9$
vertical $\parallel$ vertical $\Rightarrow$ **parallel**

(17) $y=-7$
$x=-1$
horizontal $\perp$ vertical $\Rightarrow$ **perpendicular**

(18) $y=2x+1$
$y=2$
slope 2 and horizontal (slope 0) $\Rightarrow$ **neither**

(19) $y=2x+1$
$x=2$
slope 2 and vertical (undef slope) $\Rightarrow$ **neither**

Write equations.

(20) A line parallel to $x=3$ through $(-1,4)$.
$x=3 \Rightarrow$ vertical $\Rightarrow$ vertical is $\parallel$ vertical
vertical $\Rightarrow$ x-x coord $\Rightarrow$ **$x=-1$**

(21) A line perpendicular to $x=3$ through $(-1,4)$
$x=3 \Rightarrow$ vertical $\Rightarrow$ horizontal is $\perp$ to vertical
$\Rightarrow$ horizontal $\Rightarrow$ $y=$ ycoord $\Rightarrow$ **$y=4$**.

cont'

(22) A line parallel to $y=3$ through $(-1,4)$.

$y=3 \Rightarrow$ horizontal $\Rightarrow$ parallel to horizontal is horizontal $\Rightarrow y = y$ coord $\Rightarrow \boxed{y=4}$

(23) A line perpendicular to $y=3$ through $(-1,4)$.

$y=3 \Rightarrow$ horizontal $\Rightarrow$ perpendicular to horizontal is vertical $\Rightarrow x = x$ coord $\Rightarrow \boxed{x=-1}$

(24) A line parallel to a line with $m=0$, through $(8,-9)$.

$m=0 \Rightarrow$ horizontal

$\Rightarrow$ parallel to horizontal is horizontal

$\Rightarrow$ horizontal is $y = y$ coord

$\boxed{y=-9}$

(25) A line perpendicular to a line with undefined slope, through $(-7,5)$.

m undefined $\Rightarrow$ vertical

$\Rightarrow$ perpendicular to vertical is horizontal

$\Rightarrow$ horizontal is $y = y$ coord

$\boxed{y=5}$.

(25) A line perpendicular to a line w/ $m=0$, through $(-6,4)$.

$m=0 \Rightarrow$ horizontal

$\Rightarrow$ perpendicular to horizontal is vertical

$\Rightarrow$ vertical is $x = x$ coord

$\boxed{x=-6}$

(26) A line parallel to a line with undefined slope, passing through (3, −6).

undefined slope ⇒ vertical
⇒ parallel to vertical is vertical
⇒ vertical is $x =$ x coord
$\boxed{x = 3}$.

(27) A line parallel to a vertical line, through (0, 1)

vertical ⇒ parallel to vertical is vertical
⇒ vertical is $x =$ x coord
$\boxed{x = 0}$.

(28) A line perpendicular to a horizontal line, through (1, −3).

horizontal ⇒ perpendicular to horizontal is vertical
⇒ vertical is $x =$ x coord
⇒ $\boxed{x = 1}$

(29) A line perpendicular to a vertical line, through (2, 0).

vertical ⇒ perpendicular to vertical is horizontal
⇒ horizontal is $y =$ y coord
$\boxed{y = 0}$

(30) A line parallel to a horizontal line, through (2, 3).

horizontal ⇒ parallel to horizontal is horizontal
⇒ horizontal is $y =$ y coord
⇒ $\boxed{y = 3}$.

Math 45 Summary of Techniques for Graphing a Line

Begin with Option 1. If it does not apply, try Option 2. If not Option 3, go on to Option 4, and so on.

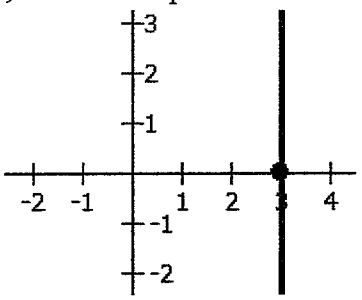
Option 1:

Ask: Is the line vertical? [Does the equation have x but no y?]

Method: Plot x-intercept and a line up and down from it.

Example 1: $x = 3$.

Plot the x-intercept at the value given, (3,0) in this example, and a line up and down from it.



Graph for Example 1

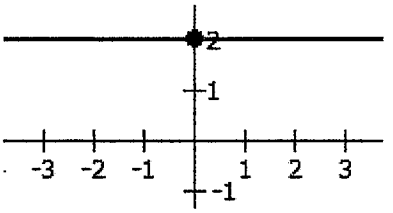
Option 2:

Ask: Is the line horizontal? [Does the equation have y but no x?]

Method: Plot the y-intercept and a line left and right from it.

Example 2: $y = 2$

Plot the y-intercept at the value given, (0,2) in example, and a line left and right from it.



Graph for Example 2

Option 3:

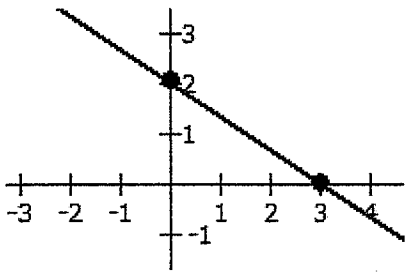
Ask: Are the x-intercept and y-intercept both integers? [Is the constant is evenly divisible by both the coefficient of the x-term and evenly divisible by the coefficient of the y-term?]

Method: Find and plot the x-intercept, find and plot the y-intercept, connect the two with a line.

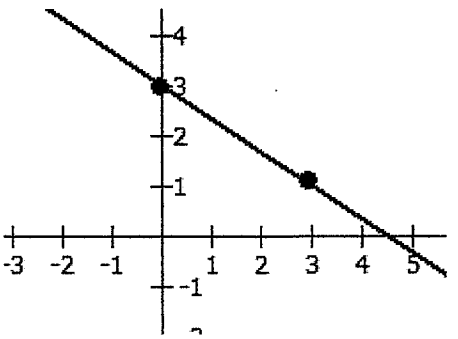
Example 3: $2x + 3y = 6$ [6 is divisible by 2 and divisible by 3]

Find the x-intercept (set y=0, solve for x), (3,0) in example, and plot it.

Find the y-intercept (set x=0, solve for y), (0,2) in example, and plot it. Connect with a line.



Graph for Example 3



Graph for Example 4

Option 4:

Ask: Is the y-intercept an integer? [Is the constant term is divisible by the y-coefficient?]

Method: Write equation in slope-intercept ($y = mx + b$) form, plot the y-intercept, use the slope.

Example 4: $2x + 3y = 9$ [9 is divisible by y-coefficient 3, but not by x-coefficient 2]

Write in slope-intercept form: $y = -\frac{2}{3}x + 3$.

Continued…

Plot the y-intercept, (0,3) in example. (continued on the back)

Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example).

From the y-intercept go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).

From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).

Option 5:

Ask: Is the x-intercept an integer? [Is the constant term is divisible by the x-coefficient?]

Method: Write equation in slope-intercept ($y = mx + b$) form, find and plot the x-intercept, use the slope.

Example 5: $2x + 3y = 4$ [4 is divisible by x-coefficient 2 but not by y-coefficient 3]

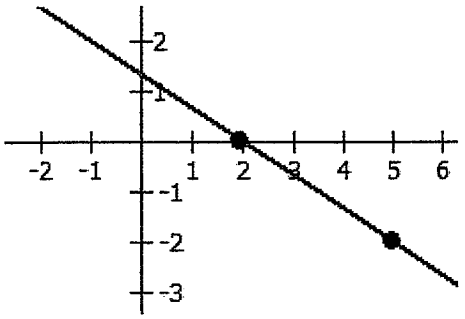
Write in slope-intercept form: $y = -\frac{2}{3}x + \frac{4}{3}$

Find and plot x-intercept (2,0).

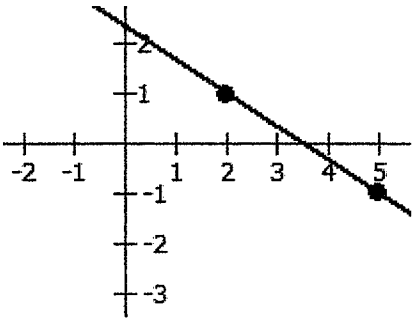
Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example)

From the x-intercept go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).

From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).



Graph for Example 5



Graph for Example 6

Option 6:

Ask: Is neither the x-intercept nor y-intercept an integer? [Is the constant term is not divisible by either the x-coefficient or the y-coefficient?]

Method: Find any point and use the slope.

Example 6: $2x + 3y = 7$. [7 is not divisible by 2 or by 3]

Choose an x-value, substitute, and solve for y, OR choose a y-value, substitute, and solve for x.]

Choosing x=0 or x=1 in this example give fractions for y. Choose x=2.

$2(2) + 3y = 7 \qquad 4 + 3y = 7 \qquad 3y = 3 \qquad y = 1$

Plot the point (in this example, (2,1))

Write the equation in slope-intercept form. ($y = -\frac{2}{3}x + \frac{7}{3}$ in this example)

Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example)

From the point go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).

From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).

Math 45 How to Write the Equation of a Line

Step 1: Recognize if the line is vertical. Write equation $x = x-coordinate$.

How to know if a line is vertical:

- It says "vertical".
- It says "slope undefined".
- It is parallel to another line with undefined slope (vertical).
- It is parallel to another line whose equation is $x = x-coordinate$ (vertical).
- It is perpendicular to a horizontal line, $y = y-coordinate$.
- It is perpendicular to a horizontal line, slope = 0.
- It is parallel to the y-axis.
- It is perpendicular to the x-axis.

Step 2: Recognize if the line is horizontal. Write equation $y = y-coordinate$.

How to know if a line is horizontal?

- It says "horizontal".
- It says "slope 0".
- It is parallel to another line with zero slope (horizontal).
- It is parallel to another line whose equation is $y = y-coordinate$ (horizontal).
- It is perpendicular to a vertical line, $x = x-coordinate$.
- It is perpendicular to a vertical line, slope undefined.
- It is parallel to the x-axis.
- It is perpendicular to the y-axis.

Step 3: Given slope and a point:

If the point is the y-intercept $(0,b)$, substitute into $y = mx + b$.

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$

Step 4: Given two points:

Find the slope using the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Substitute into the point-slope formula: $y - y_1 = m(x - x_1)$

Step 5: Given "parallel to _____" and a point.

Find the slope of the given line by writing in $y = mx + b$

Use that same slope.

Substitute slope and given point into the point-slope formula: $y - y_1 = m(x - x_1)$

Step 6: Given "perpendicular to _____" and a point.

Find the slope of the given line by writing in $y = mx + b$

Take the opposite and reciprocal of that slope to get the new slope.

Substitute new slope and given point into the point-slope formula: $y - y_1 = m(x - x_1)$